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ABSTRACT

This booklet is the second of a two-part sequence of minimum content for trigonometry. It includes sum, difference, double-angle, and half-angle formulas; Law of Sines; Law of Cosines; inverse trigonometric functions; polar coordinates; and DeMoivre's Theorem. Goals, performance objectives for each unit, a course outline, references to state-adopted texts, and teaching suggestions are given. Sample pretests and posttests are included along with an annotated list of five references. For the other booklet in this sequence, see SE 014 899. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

TRIGONOMETRY 2

5219.12

MATHEMATICS

DIVISION OF INSTRUCTION•1971

ED 067298

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

TRIGONOMETRY 2

5219.12

(EXPERIMENTAL)

Written by

David J. Johnson

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The second of a 2-quin sequence designed for the serious student who has completed Trigonometry 1 or who has encountered difficulty in Circular Functions 1. Includes sum, difference, double-angle, and half-angle formulas, Law of Sines, Law of Cosines, applications, inverse trigonometric functions, DeMoivre's Theorem.

Designed for the student who has mastered the skills and concepts of Trigonometry 1 or Circular Functions 1.

NOTE: Trigonometry 1 and 2 meet the minimum requirements for a course in trigonometry.

TABLE OF CONTENTS

OVERALL GOALS

I. The student should be familiar with the development and use of the special formulas for finding the Sine, Cosine, and Tangent of the sum of two angles, the difference of two angles, a double angle, and a half angle, as well as the development and use of reduction formulas.

II. The student should become familiar with Law of Sines and the Law of Cosines, and how to apply these to solve triangles.

III. The student should become familiar with the inverse of the trigonometric functions and their graphs.

IV. The student should be introduced to polar coordinates, the polar form of complex numbers, and DeMoivre's Theorem regarding raising a complex number to a power.

KEY TO STATE ADOPTED TEXTS

for

TRIGONOMETRY 2

D(II) Dolciani, Mary P; Wooton, William; Beckenbach, Edwin F. and Sharron, Sidney. Modern School Mathematics, Algebra 2 and Trigonometry. Atlanta: Houghton Mifflin Company, 1968.

DRO Drooyan, Irving and Hadel, Walter. Trigonometry, An Analytic Approach. New York: The Macmillan Company, 1967.

PA(II) Pearson, Helen R. and Allen, Frank B., Book Two, Modern Algebra, A Logical Approach, Including Trigonometry. Boston: Ginn and Company, 1966.

WOO Wooten, William; Beckenbach, Edwin F.; and Dolciani, Mary P.. Modern Trigonometry. Atlanta: Houghton Mifflin Company, 1966.

PERFORMANCE OBJECTIVES

The student will:

I. 1. Know and be able to show the development of each of the following formulas for sine, cosine, and tangent:

   a. The sum of two angles.

   b. The difference of two angles.

   c. The double angle.

   d. The half angle.

2. Show the development of, and use the reduction formulas for sine, cosine, and tangent.

3. Show some ability in using these formulas to prove trigonometric identities.

4. Show the use of reduction formulas in determining fuctional values of given angles.

II. 1. Know the Law of Sines and show its development.

2. Know the Law of Cosines and show its development.

3. Solve any given triangle when furnished with one of the following sets of information:

   a. The length of two sides and the measure of an angle.

   b. The length of one side and the measure of two angles.

   c. The length of three sides.

4. Show some proficiency in applying the solution of triangles to solving practical problems.

III. 1. Read both of the notations often used for the inverse relations of trigonometric functions.

2. State the inverse of any of the trigonometric functions.

3. Evaluate the inverse of any one of the trigonometric functions for a given angle.

4. Graph and recognize the graph, of any of the inverses of trigonometric functions.

IV. 1. Determine, with the use of instruments, the polar coordinates of any point in the plane.

2. Change Cartesian coordinates to polar coordinates and polar coordinates to Cartesian coordinates.

3. Change any complex number to polar form and any polar form of a complex number to standard form.

4. Use DeMoivre's Theorem to expand any complex number to an integral power or determine the integral roots of a complex number.

COURSE OUTLINE

I. Special Formulas for Determining the Sine, Cosine, and Tangent of the Sum or Difference of Two Angles, of a Double Angle, and of a Half Angle.

A. Determining odd and even integers.

B. Determining the cosine of the difference and sum of two angles.

1. Application of the distance formula to develop the formula for the cosine of the difference of two angles.

$\cos (X-Y) = \cos X \cos Y - \sin X \sin Y$

2. Using substitution to develop the formula for cosine of the sum of two angles.

$\cos (X + Y) = \cos X \cos Y - \sin X \sin Y$

3. Cosine reduction formulas.

C. Determining the sine of the difference or the sum of two angles and the sine reduction formulas.

D. Determining the tangent of the difference or the sum of two angles and the tangent reduction formulas.

E. Determining the sine, cosine, and tangent of a double angle.

F. Determining the sine, cosine, and tangent of a half angle.

G. Proving identities that involve the preceding special formulas.

H. Solving equations that involve the preceding special formulas.

II. The Law of Sines and the Law of Cosines

A. Solving right triangles:

1. When an acute angle and a side is known.

2. When two sides are known.

B. The Law of Cosines:

1. Determination of the Law of Cosines using the Pythagorean Theorem.

2. Applying the Law of Cosines to solve triangles.

C. The Law of Sines:

1. Determining the formulas for finding the area of a triangle when given two sides and the included angle.

2. Determining the Law of Sines.

3. Applying the Law of Sines to solve triangles.

4. Consideration of ambiguous cases.

D. Practical problems that involve solving triangles.

III. The Inverse of Trigonometric Functions

A. Reviewing the Inverse of an Algebriac function.

B. Determining the inverses of trigonometric functions:

1. The symbols and notation denoting inverses of Trigonometry functions.

2. Determining values of inverses of trigonometric functions.

3. Graphing inverses of trigonometric functions.

C. Proving identities that involve inverses of trigonometric functions.

D. Solving equations that involve inverses of trigonometric functions.

IV. Polar Coordinates and DeMoivre's Theorem

A. Definitions of a Polar Coordinate.

B. The Polar form of Complex Numbers.

C. DeMoivre's Theorem.

REFERENCES

OUTLINE	TOPIC	TEXT	DAYS
I A,B, C,D	The Sine, Cosine, and Tangent of a Sum or a Difference of Angles	D(II); pp. 534-549 Dro; ch 2, pp. 51-75 P(II); pp. 541-550 Woo; pp. 56-71 pp. 78-91	10
I E,F G,H	The Sine, Cosine, and Tangent of a Double Angle or a Half Angle	D(II); pp. 550-555 Dro; ch 5, pp. 148-165 P(II); pp. 550-556 Woo; pp. 78-91	5
II A	Solving Right Tri-Angles	D(II); pp 518-523 Dro; ch 6, pp. 168-173 P(II); pp. 488-494 Woo; pp. 187-192	3
II B,C D	The Laws of Sines and the Laws of Cosines	D(II); pp. 556-566 Dro; ch 6, pp. 173-184 P(II); pp. 449-512 Woo; pp. 192-203	8
III A,B C,D	The Inverses of Trigonometric Functions	D(II); pp. 568-579 Dro; ch 3, pp. 111-117 P(II); 568-578 Woo; pp. 134-148	6
IV A,B C,D	Polar Coordinates and DeMoivre's Theorem	D(II); pp. 580-591 Dro; ch 7, pp. 221-268 P(II); pp. 621-633 Woo; pp. 240-247 pp. 274-281	8

SUGGESTED STRATEGIES

1. Note that the number of scheduled days is 40, leaving a leeway of 5 days for evaluation and loss of class time.

2. In the objectives, be sure to note the words "shall know" and "shall show". Formal proofs are not emphasized in this course. The word "show" means that the student can indicate an understanding of the development, meaning, and use of the formulas, but is not required to formally prove these formulas.

3. The student is not required to know all of the reduction formulas (objective I, D) from memory, but should be able to develop any one of them as an identity test item.

4. In this course, practical uses should be stressed as much as possible. Therefore, solving triangles (objective II) should become an interesting emphasized unit.

5. The student should be made aware that the inverses of trigonometric functions are relations and not functions unless limited to a specified interval.

6. Polar coordinates are only introduced in this course. The teacher should be careful not to spend too much time on this topic. This topic occurs near the end of the suggested sequence and the amount of time remaining will determine how much can be done with polar coordinates.

7. Teachers should remember that some students will have not been introduced to complex numbers. Therefore, the presentation of Unit IV will vary, depending upon the classes background. Extra time may be needed for this unit.

8. Solving right triangles is not dealt with as a separate topic within "P". This topic can be drawn from the pages of reference, or taught as part of the applications of the Law of Sines and Law of Cosines.

9. "W" does introduce complex numbers prior to discussing polar forms of complex numbers and DeMoivre's Theorem. If the teacher is not using this text, he

might want to make use of this section in "W" for those students who need it.

10. No part of this course outline should be omitted. Of course, it can be expanded particularly in the area of practical applications, if time permits. But, be conscious of the time, so that the course outline and objectives may be completed.

SAMPLE PRE - TEST

Change the following degree measures into equivalent radian measure:

1) $30^\circ =$ 2) $225^\circ =$ 3) $150^\circ =$

Change the following radian measures into equivalent degree measure:

4) $\frac{\pi}{4}^R$ 5) $\frac{23\pi}{12}^R$ 6) $\frac{-7\pi}{6}^R$

Given that the $\sin\alpha = \frac{5}{13}$, determine each of the following:

7) $\cos\alpha =$ 8) $\tan\alpha =$ 9) $\cot\alpha =$ 10) $\csc\alpha =$

Determine the rational value of y in each of the following:

11) $y = \tan 135^\circ$ 12) $y = \cot \frac{5\pi}{4}$ 13) $y = \csc 0^\circ$

Sketch the graph of each of the following:

14) $y = \sin\alpha$ 15) $y = \tan x$ 16) $y = 3 + \sin 2x$

Solve the following equations for x:

17) $2\tan x - 2\sqrt{3} = 0$ 18) $\cos 3x = \frac{1}{2}$

Using tables, evaluate each of the following:

19) $\log .00058 =$ 20) antilog $3.42 =$

21) $\sin 89^\circ 10' =$ 22) $\sec 215^\circ 30' =$

Using tables, compute the following to 3 significant digits:

23) $\frac{(-11.011)(.953)}{(1.72)} =$

(Sample Pre - Test)
Con't.

Prove the following identities:

24) $\tan^2 x + 1 = \sec^2 x$

25) $(\tan x)(\sin x + \cot x \cos x) = \sec x$

ANSWER KEY

1. $\frac{\pi}{6}$

2. $\frac{5\pi}{4}$

3. $\frac{5\pi}{6}$

4. $45°$

5. $345°$

6. $-210°$

7. $\frac{12}{13}$

8. $\frac{5}{12}$

9. $\frac{12}{5}$

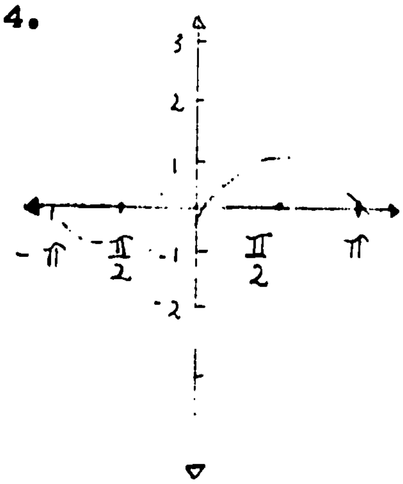
10. $\frac{13}{5}$

11. -1

12. 1

13. undefined

14.



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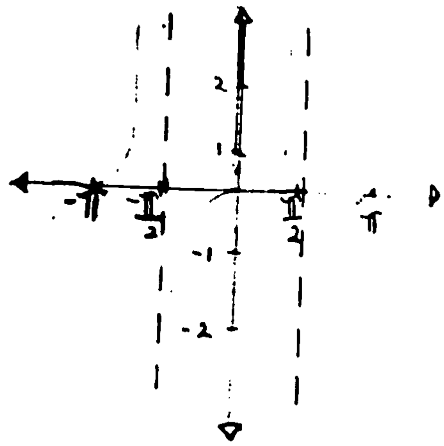

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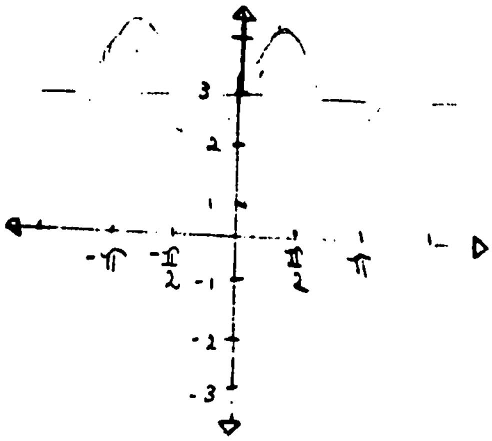


17. $\{60°, 240°\}$ or $\{\frac{\pi}{3}, \frac{4\pi}{3}\}$

18. $\{20°, 100°, 140°, 220°\}$

19. $6.7634 - 10$

20. 2,600

21. .9999

22. 1.228

23. 6100

(Answer Key)
Con't.

24. $\tan^2 x + 1 = \sec^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

25. $\tan x\,(\sin x + \cot x \cos x) = \sec x$

$$\frac{\sin x}{\cos x}\left(\sin x + \frac{\cos x}{\sin x}\cos x\right) = \frac{1}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

SAMPLE POST TEST

OBJECTIVE		TEST ITEM
I	1.	1. Complete the following formulas:
		a. $\sin(x - y) =$
		b. $\tan(x + y) =$
		c. $\cos 2x =$
		d. $\tan \frac{x}{2} =$
		e. $\cos x \cos y - \sin x \sin y =$
		f. $\pm\sqrt{\frac{1 - \cos x}{2}} =$
	2.	2. Express each of the following functions as a function of α.
		a. $\cos(\alpha + 180^\circ)$
		b. $\tan(\pi - \alpha)$
		c. $\sin\left(\alpha - \frac{3\pi}{2}\right)$
		d. $\tan(\alpha + 270^\circ)$
	3.	3. Prove the following:
		a. $\cos(x + y)\cos(x - y) - \sin(x + y)$ $\sin(x - y) = \cos^2 - \sin^2 x$
		b. $\sin(x + y)\cos(x - y) - \cos(x + y)$ $\sin(x - y) = 2 \sin y \cos y$
	4.	4. Determine the value of the following by using the reduction formulas.
		a. $\sin 105^\circ$
		b. $\tan 75^\circ$

(Sample Post Test)
Con't.

OBJECTIVE	TEST ITEM
II. 1. 2.	5. State each of the following: a. Law of Cosines b. Law of Sines
3.	6. Solve the following trianges completely. Approximate angles to the nearest degree. a. $m(C) = 90^\circ$, $b = 5$, $c = 13$ b. $a = 20$, $b = 34$, $c = 48$
4.	7. Solve the following: a. Two airplanes leave the airport at the same time, one flying on a straight course at 400 miles per hour and the other on a straight course at 480 miles per hour. If they are 1400 miles apart after 3 hours, what is the measure of the angle between their lines of flight? b. The angle of depression of a ship from the top of a 75 foot lighthouse measures 21°. How many feet is the ship from the base of the lighthouse?
III. 2.	8. Evaluate each of the following, using tables to approximate if necessary: a. $\sin^{-1} \frac{1}{2} =$ b. Arctan $\sqrt{2}$ c. $\cos \left[\text{Arcsin } -\frac{3}{5} \right] =$ d. $\sin^{-1} \left[\cos \frac{\pi}{4} \right] =$

(Sample Post Test)
Con't.

OBJECTIVE	TEST ITEM
1. 3.	9. State the inverse of each of the following functions: a. $y = \sin x$ b. $y = \tan x$
4.	10. Graph the following: a. $y = \arctan x$ b. $y = \sin^{-1} x$
IV. 1. 2.	11. Find a pair of polar coordinates with the angles less than 360° for the following points described by Cartesian coordinates: a. $(3, 3\sqrt{3})$ b. $(3, -4)$
	12. Find the Cartesian coordinates for the following points described by polar coordinates: a. $(3, 180^\circ)$ b. $(4, \frac{\pi}{2})$
3.	13. State the following complex numbers in polar form: a. i b. $-1 + i\sqrt{3}$
	14. State the following polar form of complex numbers in standard form: a. $3(\cos 120^\circ + i \sin 120^\circ)$ b. $4\left[\cos\left(\frac{-\pi}{3}\right) + i \sin\left(\frac{-\pi}{3}\right)\right]$

(Sample Post Test)
Con't.

OBJECTIVE	TEST ITEM
4.	15. State the following in the form of $a + bi$ using DeMoivre's Theorem. a. $(1 + i\sqrt{3})^4$ b. $(2 - 2i)^{-2}$
	16. What are the four roots of: $16(\cos 120^\circ + i \sin 120^\circ)$?

POST TEST ANSWER KEY

1. a. $\sin x \cos y - \cos x \sin y$

   b. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

   c. $2 \cos^2 x - 1$

   d. $\pm\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

   e. $\cos (x + y)$

   f. $\sin \dfrac{x}{2}$

2. a. $-\cos\alpha$ b. $-\tan\alpha$ c. $\cos\alpha$ d. $-\cot\alpha$

3. a. $(\cos x \cos y - \sin x \sin y) \cdot (\cos x \cos y + \sin x \sin y) -$
   $(\sin x \cos y + \cos x \sin y) \cdot (\sin x \cos y - \cos x \sin y) =$
   $\cos^2 x - \sin^2 x$

   $\cos^2 x \cos^2 y - \sin^2 x \sin^2 y - \sin^2 x \cos^2 y + \cos^2 x \sin^2 y =$
   $\cos^2 x - \sin^2 x$

   $(\cos^2 x - \sin^2 x)(\cos^2 y + \sin^2 y) = \cos^2 x - \sin^2 x$

   $\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$

   b. $(\sin x \cos y + \cos x \sin y) \cdot (\cos x \cos y + \sin x \sin y) -$
   $(\cos x \cos y - \sin x \sin y) \cdot (\sin x \cos y - \cos x \sin y) =$
   $2 \sin y \cos y$

   $\sin x \cos x \cos^2 y + \cos^2 x \sin y \cos y + \sin^2 x \sin y \cos y +$
   $\sin x \cos x \sin^2 y - \sin x \cos x \cos^2 y - \sin x \cos x \sin^2 y +$
   $\sin^2 x \sin y \cos y + \cos^2 x \sin y \cos y = 2 \sin y \cos y$

   $(\sin y \cos y)(2 \cos^2 x + 2 \sin^2 x) = 2 \sin y \cos y$

   $2 \sin y \cos y = 2 \sin y \cos y$

4. a. $\dfrac{1}{4}(\sqrt{6} + \sqrt{2})$

   b. $\dfrac{3 + \sqrt{3}}{3 - \sqrt{3}}$

(Post Test Answer Key)
Con't.

5. a. $a^2 = b^2 + c^2 - 2\,bc \cos A$

b. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

6. a. $a = 12$ $m(A) \approx 67°$ $m(B) \approx 23°$

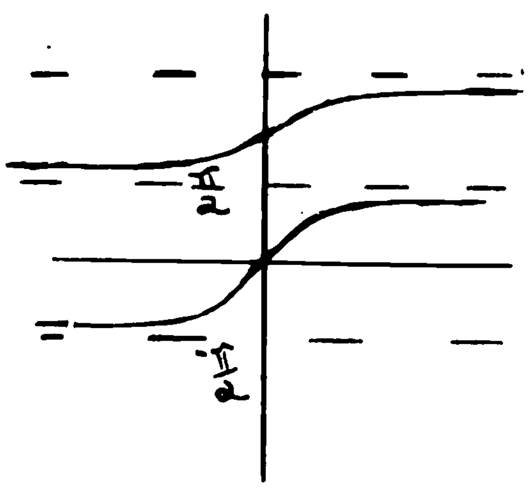
b. $m(A) \approx 20°$ $m(B) \approx 36°$ $m(C) \approx 123°$

7. a. $63°$ b. 195 ft.

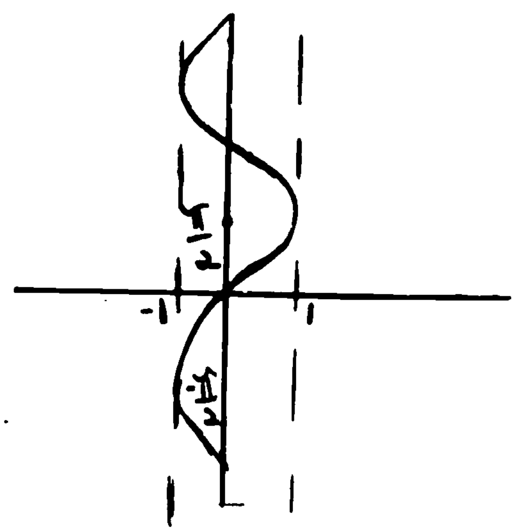
8. a. $30°$ b. $55°$ c. $\frac{4}{5}$ d. $\frac{\pi}{4}$

9. a. $x = \sin^{-1} y$ b. $x = \arctan y$

10. a.



b.



11. a. $(6, 30°)$ b. $(5, 127°)$

12. a. $(-3, 0)$ b. $(0, 4)$

13. a. $1\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$ b. $2(\cos 120° + i \sin 120°)$

14. a. $-\frac{3}{2} + \frac{3}{2}\sqrt{3}\,i$ b. $2 + 2\sqrt{3}\,i$

(Post Test Answer Key)
Con't.

15. a. $-8 - 8\sqrt{3}\,i$ b. $\frac{1}{8}i$

16. $\sqrt{3} + i$, $-1 + \sqrt{3}\,i$, $-\sqrt{3} - i$, $1 - \sqrt{3}\,i$

ANNOTATED BIBLIOGRAPHY

for

TRIGONOMETRY 2

1. Dressler, Isidore and Rich, Barnett. Review Text in Eleventh Year Mathematics. New York: Amsco School Publications, Inc., 1960.
(This is a more traditional approach. However, it is a good concise and brief review. Excellent resource for text items.)

2. Johnson, Richard E. ; McCoy, Neal H. and O'Neill, Anne F.. Introduction to Mathematical Analysis. New York: Holt, Rinehart and Winston, Inc., 1962.
(Chapter 2 of this text is a good review for the teacher primarily. Many texts in Mathematical Analysis do have good concise reviews of Trigonometry.)

3. Smith, Rolland R.; Lankford, Francis G., Jr. and Payne, Joseph N., Contemporary Algebra, Book Two. Atlanta: Harcourt, Brace and World, Inc., 1963.
(Chapter 14 of this text is not thorough in many elementary topics, but does stress the practical sides of Trigonometry. Some very good practical problems.)

4. Vannatta, Glen D.; Carnahan, Walter H. and Fawcett, Harold P.. Advanced High School Mathematics. Columbus, Ohio: E. Merrill Publishing Company, 1965.
(Chapters 4,5,6,7, and 8 cover all the topics of Trigonometry I and II. It is an easily read text.)

5. Welchons, A. M.; Krickenberger, W. R. and Pearson, Helen R.. Modern Trigonometry. Atlanta, Ginn and Company, 1962.
(Many schools will have copies of this text since it was once a state adopted text. It could be used as resource and help to a student simply as another approach to a topic.)